MMATH MIDTERM EXAMINATION ALGEBRAIC GEOMETRY

Attempt all questions. Assume that the base field k, in all questions below, is algebraically closed. Total: 50 marks. Time: 3 hours.

(1) Consider the set $X \subset \mathbb{A}_k^3$ be the union of x, y and z axes. Calculate $I(X)$. (8 marks)
(2) Let $X = Z(y^2 - x^3 + x) \subset \mathbb{A}_k^2$. Is X irreducible? Justify your answer. (8 marks)
(3) What is the ring of regular functions on the open set $\mathbb{A}_k^2 - \{(0,0)\} \subset \mathbb{A}_k^2$? Justify your answer. (8 marks)
(4) Describe all morphisms from $\mathbb{P}_k^n$ to $\mathbb{A}_k^m$ (for any two positive integers n and m). (8 marks)
(5) Define a Noetherian topological space. Prove that any open cover of a Noetherian topological space has a finite subcover. (8 marks)
(6) Using the sheaf of regular functions on $\mathbb{A}^1 - \{0\}$, prove that it is isomorphic to an affine variety. (10 marks)